Making Sense of Number, K-10

Getting to know your students so you can support the development of their mathematical understanding

A Sampler of Frameworks for Learning

A Framework for Structuring Learning Experiences

If mathematical proficiency is our desired outcome, and we believe that having a sense of number is explicitly linked to mathematical proficiency, then we need to clarify how we are going to structure learning experiences for mathematics teaching and learning. Number sense is our ultimate goal for our students, which allows them to make sense of the world through a numerate lens. We have synthesized our understanding of these principles, dimensions, and practices and embedded them within the structure we developed to help you make connections to everyday teaching and learning (see Figure 1.8).

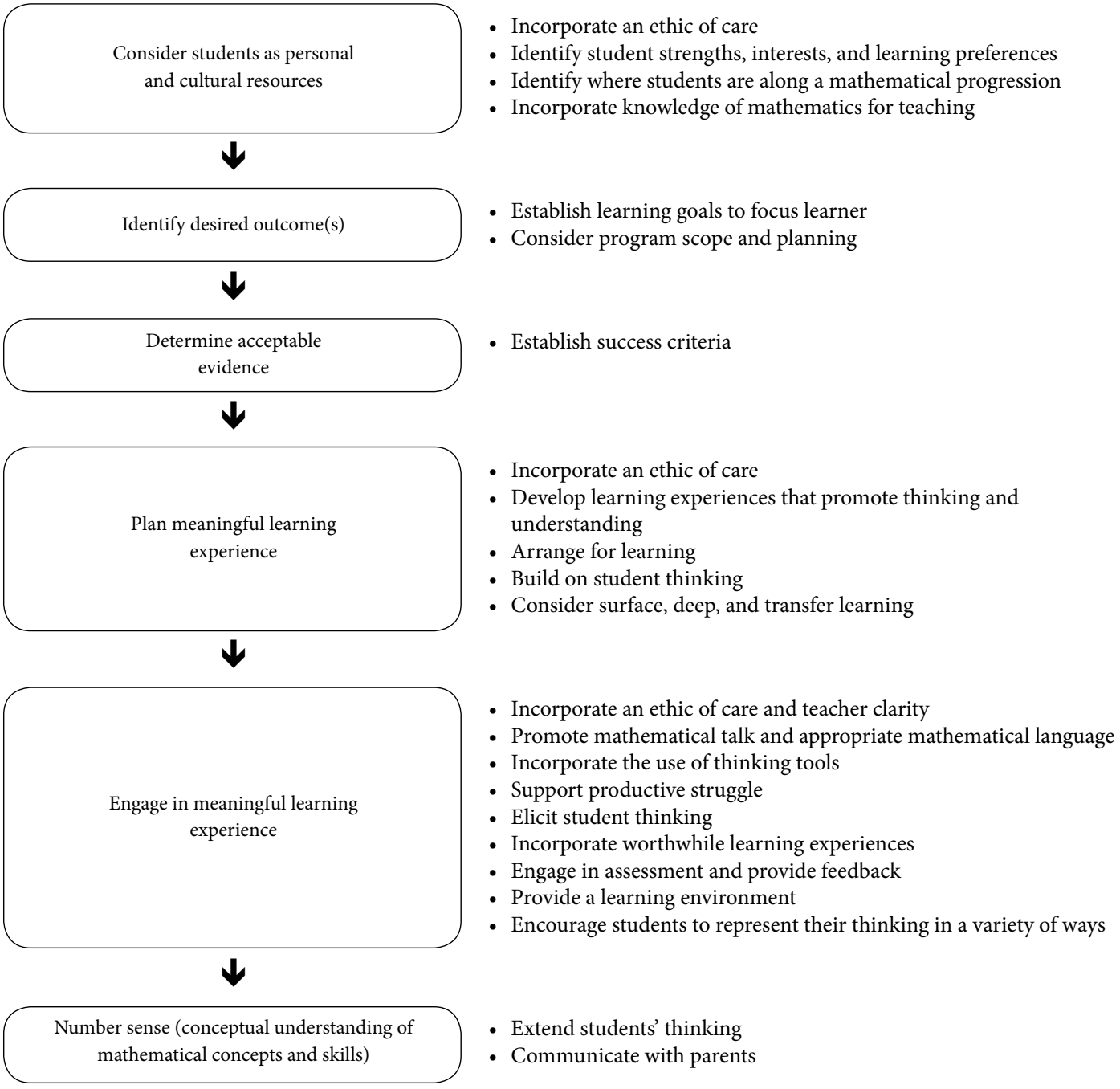

Figure 1.8: *Framework for Structuring Learning Experiences and Actions that Create Meaningful Learning*

Supporting Mathematical Thinking

Thinking is an integral component of mathematics curricula and central to the teaching and learning of mathematics. Mathematical thinking includes mental actions and specific behaviors that students rely on as they engage in learning experiences designed to help them better understand mathematical concepts and skills and the role mathematics plays in the world around them. It is not thinking about the subject matter of mathematics, but rather a way of thinking about mathematical concepts and skills. Teaching through thinking supports learning with understanding, enabling students to solve the new kinds of problems they will inevitably face in the future.

SUPPORTING MATHEMATICAL THINKING

Thinking Skills	Bigger Ideas
Inferring and interpreting	• Making meaning and reaching an interpretation involves identifying stated and implied information and ideas. • Drawing meaning from explicit and implicit information requires use of prior knowledge and experiences. • Interpreting information and drawing a conclusion about it involves using reasoning to think about evidence, both stated and implied.
Making connections	• Understanding of texts, information, concepts, procedures, and skills is deepened through the use of prior knowledge, experiences, and opinions. • Gaining deeper meaning of concepts and skills occurs when relationships are identified and how they support understanding is explained. • Understanding is extended by comparing and contrasting information and ideas to one's own knowledge and experiences.
Analyzing	• Breaking down information involves identifying the parts of elements, and describing their purpose and function and how they contribute to meaning. • Determining how the parts or elements are connected to one another can be achieved by classifying, comparing, and contrasting information and ideas. • Inferring the relationship between the parts relies on using evidence to support generalizations, conclusions, and assumptions.
Evaluating	• Forming and defending opinions requires making judgments about information and ideas. • Justifying reasons for an informed decision involves using established criteria that help determine the validity and quality of information and ideas. • Assessing something's effectiveness involves using a set of criteria to draw conclusions about information, evidence, and ideas.
Synthesizing	• Combining and integrating ideas will lead to the creation of a new understanding. • Current understanding evolves and changes as more information and experiences are acquired. • Identifying when an understanding shifts and changes supports reasoning and proving.
Reasoning and proving	• Drawing conclusions for a justification involves inferring hypotheses and making conjectures. • Justifying one's thinking requires providing evidence that is reasonable and valid—that is, effective. • Being able to explain why conclusions and arguments are logical depends on evaluating the validity of proof.
Reflecting	• Thinking about one's own thought processes to acquire a deeper understanding of concepts allows monitoring of one's learning. • Analyzing and evaluating strategies used to improve learning enables assessment of one's understanding. • Thinking about how to expand knowledge or extend ability to make connections and gain understanding allows one to transfer learning to new contexts.

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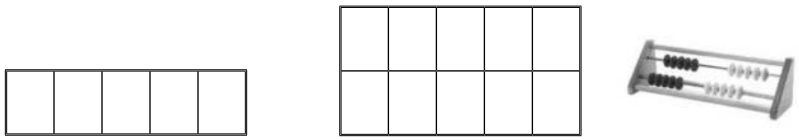
Using Models to Support Understanding

Models are a powerful way to explore numbers and help students make connections between representations. As students develop strategies and big ideas over time, it is important to introduce models that will help students extend and consolidate their understanding. Exploring numbers in many contexts plays a central role in the teaching and learning of number sense. How students are introduced to numbers helps them to clarify their understanding, make connections, and gain flexibility with numbers. For example, the models in Figure 3.2 can be used to help support students' understanding of early number relationships.

Spatial relationships: dot arrangements on various classroom materials (e.g., number cubes, dot plates)

1 or 2 more/less: number paths

Benchmarks of 5 and 10: five- and ten-frames and rekenreks (arithmetic racks)



Parts/whole relationships: all of the above

Figure 3.2: *Models to Support Understanding of Number Relationships*

Introducing models in context is a powerful strategy to help students make sense of the model. The eventual goal is to develop models in such a way that students use them as powerful tools for thinking. With a growing familiarity with models, students will begin to use them to demonstrate their own thinking, eventually leading to students creating a mental version of the model that can be accessed as a tool for thinking as needed (see Figure 3.4).

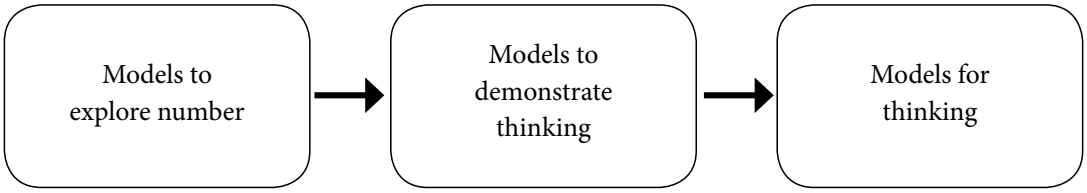


Figure 3.4: *Using Models*

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Making Generalizations

One of our goals as teachers is to carefully introduce our students to mathematical situations that will help them to develop deep conceptual understandings. As students develop sophisticated strategies and big ideas, we want to help students see the connections between what they are currently exploring and their previous experiences, and we want to help them formalize the relationships that are inherent in the number system. When students are constructing concepts, they make conjectures about mathematical situations but often require educator intervention to test and formalize a generalization about a mathematical relationship. At each step of the way, it is helpful to have a bank of prompts that will help make these relationships visible. Figure 6.1 outlines the process students follow to make generalizations, as well as sample prompts teachers can use during this process.

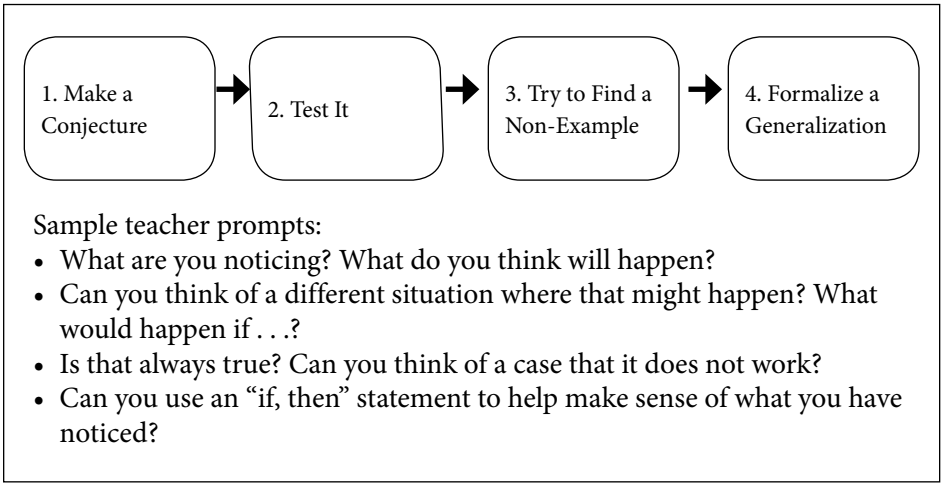

Figure 5.1: *Making Generalizations*

As students develop the four operations over time, they come to understand how the parts of a number sentence relate to each other. As they move through a progression for operational sense, we can help students formalize these relationships by supporting the making of generalizations.

STRATEGIES TO DEVELOP NUMBER SENSE	
Generalization	**Sample Learning Experience**
If I add 1 to an addend and the other remains the same, then I add 1 to the sum.	How does knowing 5 + 5 = 10 help you to solve 5 + 6?
If I add the same amount to both the minuend and the subtrahend, the difference remains the same.	The difference between Harman and Simran's ages is 4. How old might they be? What will be the difference in their ages in 5, 10, 25, and 1,000 years?
If I increase the multiplicand by 1 and the multiplier remains the same, then I add the multiplier to the product.	5×4 8×4 5×5 8×5 5×6 8×6
If I double the dividend and the divisor remains the same, then the quotient is doubled.	How does knowing 64 ÷ 8 = 8 help you to solve 128 ÷ 8?

Making Sense of Representing

Students engage in reasoning and use representations to communicate their mathematical thinking and understanding of mathematical concepts and skills. When they are communicating, students are making their thinking visible and providing opportunities to build upon their own thinking as well as the thinking of others. Representing, simply stated, is about capturing a mathematical thought. Once captured, representations can then be used as a springboard to think more deeply.

Representing mathematical thoughts plays an important role in teaching and learning. Students have the important role of reflecting on their mathematical thoughts and making their understanding visible. How students choose to represent their thoughts helps to clarify their understanding and tells us different things. For example, a student may represent their thinking of the number 30 by using an array (see Figure 6.1). The representation helps us understand that a student is thinking of equal groups in a rectangular fashion. The representation also tells us that a student is able to think of a number as the product of two numbers.

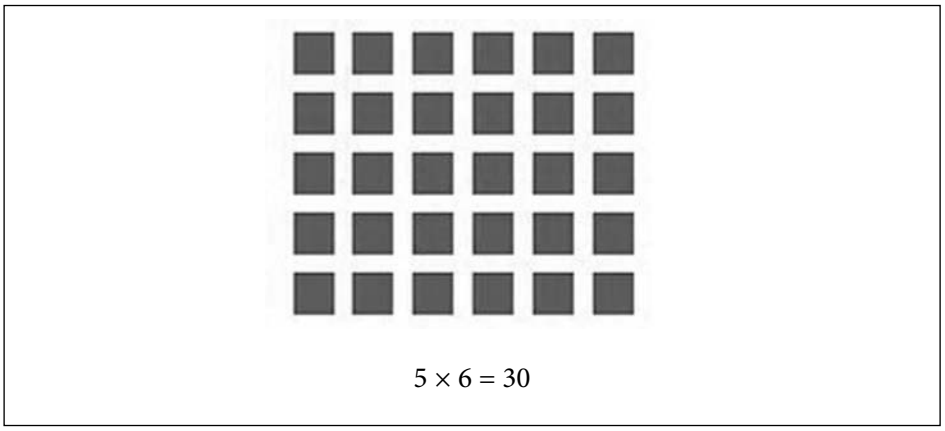

Figure 6.1: *Array Representing 30*

Another student may represent their thinking of 30 using a number line (see Figure 6.2). This representation helps us understand that a student has an understanding of the use of benchmark numbers to help order and compare numbers.

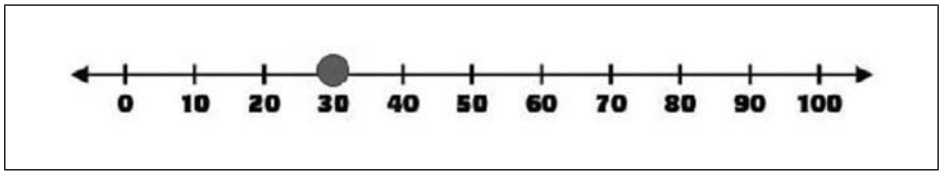

Figure 6.2: *Number Line Representing 30*

Both representations clarify the student's thinking about the number 30 and provide opportunities for students to make connections between the two representations. By posing further questions, such as, "How could you use a number line to show the mathematical thought represented in the array?" or "How could you use the number line to represent the product of 5 and 6?," students are able to make connections and deepen their understanding of number. We therefore have the important role of creating learning experiences where students have an opportunity to represent their thinking and understanding about a mathematical idea in a number of ways. This includes encouraging students to represent their mathematical thoughts in ways that make sense to them. How a student represents their thoughts helps us determine where they are along a mathematical learning progression and plan next steps based on this understanding.

Reflecting on Professional Learning

Parallel to student learning is teacher learning. Professional learning experiences designed to deepen our knowledge and understanding of how effective practices promote meaningful learning experiences are integral to teaching and learning. Although knowledge and understanding of effective practices are necessary, they are not sufficient. It is when we apply and transfer our knowledge as part of our daily practices that we foster mathematical learning experiences that enhance students' mathematical thinking and develop students' sense of number.

Seven Key Concepts

By emphasizing seven key concepts as part of the consolidation process, we can transfer and apply our knowledge and understanding of effective practices to support the teaching and learning of mathematics as part of our daily practice. Each key concept is addressed as a purpose statement to incorporate teacher actions and intended student outcomes. Note that each purpose statement was developed by first considering students' learning needs—gathering and analyzing evidence from a variety of sources—before selecting actions to address those needs. The goal is to provide examples of purpose statements or theories of action that we can use as part of our personal professional learning journey.

Key concept #1: If we engage in professional learning—whether informal or formal—that shifts our thinking and practice to address an identified student learning need, then students will be able to learn something new in a different way.

Key concept #2: If we engage in professional noticing to gather evidence of student thinking and learning, then students will be inclined to make their thinking visible.

Key concept #3: If we develop an understanding of and use learning progressions to purposely plan for meaningful learning experiences, then all students will be able to engage with mathematics.

Key concept #4: If we engage in reflective practice to make connections between our thinking and our students' thinking, then students will see themselves as part of the curriculum.

Key concept #5: If we develop and use learning intentions to align meaningful learning experiences to the "important stuff," then students will view mathematics as more than just rules and procedures.

Key concept #6: If we plan for and implement learning experiences that provide students with opportunities to engage in mathematical thinking, then students will develop an understanding of mathematical concepts.

Key concept #7: If we plan for learning experiences that are purposefully designed to elicit mathematical thinking, then students will better understand the role of mathematics in the world around them.

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Student Profile Template

STUDENT PROFILE

Name: _____ Age: _____

Grade: _____

School: _____

Number of Credits Accumulated: _____ Date: _____

Sources of Information

(Identify sources of information and assessments to be conducted.
Check box and note date when a source has been reviewed or a new assessment completed.)

Review of OSR, including previous report cards _____
Consultation with parents _____
Consultation with previous and current teachers _____
Consultation with support team _____
Classroom observation checklist _____
Educational assessments (e.g., pretests related to particular curriculum expectations) _____

Interest and/or learning style inventory _____
Work samples, assignments, projects _____
Portfolios _____
Teacher-student conferences _____
Peer and self assessments _____
Other (specify) _____

Findings from Information Sources and Assessments – Strengths and Areas of Need

Current achievement levels, learning skills/work habits and readiness to learn	Learning styles/preferences and needs, interests, social/emotional strengths and needs	Other relevant information

Assessment and Instruction

Considerations for Instructional Strategies	Considerations for Assessments	Available Resources and Supports

Source: Ontario Ministry of Education, 2013; http://www.edu.gov.on.ca/eng/general/elemsec/speced/LearningforAll2013.pdf

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Designed to enhance your professional learning, this practical book shows you how to notice, interpret, confirm, and respond to student thinking. It offers an effective approach to finding out what your students already know so that you can structure learning experiences that help them make sense of number.

Making Sense of Number outlines a range of learning experiences that will help students build on their mathematical understanding. It is organized around key number concepts—quantity, counting, relating, and representing—and across mathematical strands (patterning and algebra, number and operations, measurement, geometry, and data and probability).

This timely book offers learning experiences that support students in their mathematical journey. Each learning experience includes

- important features associated with each of the key number concepts
- a developmental progression for each number concept
- thinking stems and prompting questions to support student understanding
- descriptions of specific student thinking and teacher thinking
- opportunities for reflection to enhance professional learning

Ideal for new and experienced teachers, ***Making Sense of Number*** shows you how to give K–10 students the support they need as they develop confidence in their number sense.

Mary Fiore is a curriculum and instructional leader who works collaboratively with educators to explore innovative ways to support the teaching and learning of mathematics. Mary has contributed to a variety of professional learning resources that focus on thinking differently about mathematics and making students' mathematical thinking visible. She is the co-author of *Moving Math* and *The Four Roles of the Numerate Learner*. As part of her commitment and passion toward the teaching and learning of mathematics, Mary engages educators in professional learning opportunities that support curriculum, instruction, assessment, and student engagement.

Ryan Tackaberry describes himself as, "Just a guy who loves teaching math." As an energetic and passionate mathematics leader in his school district, he enjoys exploring an equitable approach to teaching and learning. He has collaborated with Mary Fiore and others to create professional learning resources that support teachers with learning experiences that are rich with differentiated instruction for students in mathematics. Ryan continues to inquire about the development of student thinking over time and how the use of mathematical models supports learners to obtain a deep understanding of concepts and skills.

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